

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3

Branch: B.Tech (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) The infinite series $1 + r + r^2 + \dots + r^{n-1} + \dots$ is convergent if
 (A) $|r| < 1$ (B) $|r| > 1$ (C) $r = 1$ (D) $r < -1$
- b) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (A) $\log 2$ (B) zero (C) infinite (D) none of these
- c) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x \, dx$ is
 (A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$
- d) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then $(f_n + f_{n-2})$ is equal to _____.
 (A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$
- e) $\int_1^{\infty} \frac{1}{x^{\sqrt{2}}} \, dx$ is convergent.
 (A) True (B) False
- f) $\int \sqrt{n} \sqrt{n-1} =$ _____.
 (A) $\frac{\pi}{\cos n\pi}$ (B) $\frac{\pi}{\sec n\pi}$ (C) $\frac{\pi}{\cos ecn\pi}$ (D) $\frac{\pi}{\sin n\pi}$
- g) If $B(x, 2) = \frac{1}{3}$, then the value of $x =$ _____.
 (A) 0 (B) 1 (C) 2 (D) none of these
- h) If the power of y are even, then the curve is symmetrical about
 (A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these



- i) $\int_0^1 dx \int_0^x e^x dy$ is equal to
 (A) $e+1$ (B) $e-1$ (C) $\frac{1}{2}(e+1)$ (D) $\frac{1}{2}(e-1)$
- j) On converting into polar coordinates $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx dy$ is equal to
 (A) $\int_0^{\pi} \int_0^{2a\cos\theta} r dr d\theta$ (B) $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r dr d\theta$ (C) $\int_0^{\frac{\pi}{2}} \int_0^{2a\sin\theta} r dr d\theta$ (D) none of these
- k) The transformations $x+y=u, y=uv$ transform the area element $dy dx$ into $|J| du dv$, where $|J|$ is equal to
 (A) 1 (B) u (C) -1 (D) none of these
- l) The degree of the differential equation $3\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$ is
 (A) 1 (B) 2 (C) 3 (D) 6
- m) The solution of $\frac{dy}{dx} = e^{x+y}$ is
 (A) $e^x - e^{-y} = c$ (B) $e^x - e^y = c$ (C) $e^x + e^{-y} = c$ (D) $e^x + e^y = c$
- n) The orthogonal trajectories of the family of curve $y = cx^k$ are given by
 (A) $x^2 + ky^2 = \text{const.}$ (B) $x^2 + cy^2 = \text{const.}$ (C) $kx^2 + y^2 = \text{const.}$
 (D) $x^2 - ky^2 = \text{const.}$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$ (5)

b) Using reduction formula evaluate: $\int_0^1 x^6 \sin^{-1} x dx$ (5)

c) Prove that $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx = \frac{1}{5005}$. (4)

Q-3 Attempt all questions (14)

a) Prove that $\int_0^1 x^{q-1} \left(\log \frac{1}{x}\right)^{p-1} dx = \frac{\Gamma(p)}{q^p}$. (5)

b) Using reduction formula prove that $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left(\log 2 - \frac{1}{2}\right)$. (5)

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$. (4)

Q-4 Attempt all questions (14)



a) Change the order of integration in the integral $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$ and evaluate it. (5)

b) Examine the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2 + 1} + \dots$ for convergence using ratio test. (5)

c) Solve: $(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$ (4)

Q-5

Attempt all questions (14)

a) Solve: $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$ (5)

b) By changing into polar co-ordinates, evaluate the integral (5)

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx dy .$$

c) Using reduction formula, evaluate: $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$ (4)

Q-6

Attempt all questions (14)

a) Evaluate: $\int_0^{\infty} x^4 e^{-x^4} dx$ (5)

b) Solve: $xdy - ydx = \sqrt{x^2 + y^2} dx$ (5)

c) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ (4)

Q-7

Attempt all questions (14)

a) Trace the curve $r^2 = a^2 \cos 2\theta$. (5)

b) Evaluate: $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$ (5)

c) Find the length of the arc of the curve $y = \log \sec x$ from $x=0$ to $x = \frac{\pi}{3}$. (4)

Q-8

Attempt all questions (14)

a) Show that the volume of the spindle-shaped solid generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis is $\frac{32\pi a^3}{105}$. (5)

b) Trace the curve $y^2(2a-x) = x^3$. (5)

c) Investigate the convergence of $\int_2^5 \frac{1}{\sqrt{(x-2)}} dx$. (4)

